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Induced Chern-Simons and WZW action in Noncommutative Spacetime

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Abstract

We consider noncommutative gauge theory with Dirac or Majorana fermions in odd dimensional spacetime and compute the induced noncommutative Chern-Simons action generated at 1-loop. We observe that the Chern-Simons term induced by a Dirac fermion has a smooth limit when $\theta \to 0$, but there is a finite jump for the Chern-Simons term induced by a Majorana spinor. The induced Chern-Simons action from a Majorana spinor is nonvanishing even in the $\theta \to 0$ limit, a discontinuity that share a similar characteristic as the UV/IR singularity discovered originally by Minwalla, Raamsdonk and Seiberg. Properties of the noncommutative WZW action are also discussed.

1 Introduction

Chiral fermions in even dimensions can give rise to an anomaly [1]. Although there is no chirality in odd dimensions, there is still a similar phenomenon for fermions. It is well-known that due to radiative effects at one fermion loop, a Chern-Simons action for the gauge bosons can be generated in odd dimensions [2, 3]. This phenomenon has wide applications in particle and condensed matter physics [4].

Recently, noncommutative field theory has shown up as effective description of string theory in a certain background [5, 6, 7, 8]. The noncommutativity takes the form

$$[x^\mu, x^\nu] = i\theta^{\mu\nu}, \tag{1}$$

where $\theta^{\mu\nu}$ is a antisymmetric real constant matrix and is of dimension length squared. In the dual language, the algebra of functions is described by the Moyal product

$$(f * g)(x) = e^{i\frac{\theta^{\mu\nu}}{2}\frac{\partial}{\partial\xi^\mu}\frac{\partial}{\partial\zeta^\nu}} f(x+\xi)g(x+\zeta)|_{\xi=\zeta=0}, \tag{2}$$

which is associative, noncommutative and satisfies

$$\overline{(f * g)} = \bar{g} * \bar{f} \tag{3}$$

under complex conjugation. We note also that under integration

$$\int f * g = \int g * f = \int fg, \tag{4}$$

which is a consequence of momentum conservation. Using this $*$-product, field theory on a noncommutative spacetime can be easily formulated. One simply needs to replace the usual multiplication of functions by the Moyal product. Pioneering analysis of this kind of noncommutative field theory was performed by Filk [9]. Aspects of noncommutative field theories was further developed in [10]-[26].

In this paper we analyze gauge theory with fermions defined on a noncommutative odd dimensional spacetime and determine the corresponding induced Chern-Simons action. Since the action for the noncommutative theory is a smooth deformation of the classical action, one may natively expect to get back the usual commutative description in the limit $\theta \to 0$. We find that this is not always the case when one is in the quantum regime: for the case of a Majorana spinor, there is a jump in the induced Chern-Simons action. Singularities in θ have also been displayed in the scalar theories [16, 17, 18] and in QED [19, 20].

2 Induced Chern-Simons in odd dimensions

We will begin with noncommutative QED in (2+1)-dimensions with a 2-components massless fermion. We will take $\theta_{12} = \theta \neq 0$, and the action is given by

$$S = \int d^3x (-\frac{1}{4} F_{\mu\nu} * F^{\mu\nu} + i\bar{\psi} * \not{D} * \psi), \tag{5}$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + ig[A_\mu, A_\nu]_*$ and the covariant derivative (and hence the fermion coupling) is given by $D_\mu \psi = \partial_\mu \psi + igA_\mu * \psi$ for a Dirac spinor and $D_\mu \psi = \partial_\mu \psi + ig[A_\mu, \psi]_*$ for a Majorana spinor. These couplings reproduce the correct corresponding commutative limit, in particular a Majorana spinor is neutral in the commutative case.

The γ-matrices are given by the Pauli matrices and satisfies

$$\gamma^\mu \gamma^\nu = -g^{\mu\nu} - i\epsilon^{\mu\nu\lambda}\gamma_\lambda, \tag{6}$$

where $g^{\mu\nu} = (-, +, +)$ and $\epsilon^{012} = +1$. The action is invariant under the gauge transformation

$$\delta A_\mu = \partial_\mu \alpha - ig[\alpha, A_\mu]_*, \tag{7}$$

and $\delta\psi = -ig\alpha * \psi$ or $\delta\psi = -ig[\alpha, \psi]_*$ for a Dirac or Majorana spinor respectively.

The one fermion-loop effective action is given by

$$\begin{aligned} S_{eff}[A, m] = \quad & \frac{1}{2} \int \frac{d^3p}{(2\pi)^3} A_\mu(p) A_\nu(-p)(i\Gamma^{\mu\nu}(p)) + \\ & + \frac{1}{3} \int \frac{d^3p_1}{(2\pi)^3} \frac{d^3p_2}{(2\pi)^3} A_\mu(p_1) A_\nu(p_2) A_\lambda(-p_1 - p_2)(i\Gamma^{\mu\nu\lambda}(p_1, p_2)) \end{aligned} \tag{8}$$

for $m = 0$. The 2-point and 3-point functions $\Gamma_{\mu\nu}(p)$ and $\Gamma_{\mu\nu\lambda}(p)$ will be analyzed now.

2.1 Dirac fermions

We first consider the case of a Dirac spinor. It is crucial [19] to observe that for Dirac spinors, one only gets planar diagrams from doing the Wick contractions. Hence the 2-point and 3-point functions are

$$i\Gamma_D^{\mu\nu}(p) = g^2 \int \frac{d^3k}{(2\pi)^3} \mathrm{tr}[\gamma^\mu \frac{\not{k} - \not{p} - m}{(k-p)^2 + m^2} \gamma^\nu \frac{\not{k} - m}{k^2 + m^2}], \tag{9}$$

$$i\Gamma_D^{\mu\nu\lambda}(p_1, p_2) = -g^3 \int \frac{d^3k}{(2\pi)^3} \frac{\mathrm{tr}[\gamma^\mu(\not{k} - m)\gamma^\nu(\not{k} + \not{p}_2 - m)\gamma^\lambda(\not{k} - \not{p}_1 - m)]}{(k^2 + m^2)((k+p_2)^2 + m^2)((k-p_1)^2 + m^2)} e^{-\frac{i}{2}p_1\theta p_2}. \tag{10}$$

The only difference from the commutative case is the phase factor in (10) which depends only on the external momenta.

The effective action (8) is ultraviolet divergent and needs to be regularized. This can be achieved by the standard Pauli-Villars method:

$$S^{reg}_{eff}[A] = S_{eff}[A, m = 0] - \lim_{m\to\infty} S_{eff}[A, m]. \tag{11}$$

As in the commutative case, apart from the wanted terms that cure the divergences of (8), there are terms that remain even after sending $m \to \infty$ and these terms give rises to the induced Chern-Simons Lagrangian. One can verify that in the large m limit:

$$\begin{aligned}
\lim_{m\to\infty} i\Gamma^D_{\mu\nu}(p) &= -\frac{\Lambda}{3\pi^2} g_{\mu\nu} - \frac{i}{4\pi}\frac{m}{|m|}\epsilon_{\mu\nu\lambda}p^\lambda, && (12)\\
\lim_{m\to\infty} i\Gamma^D_{\mu\nu\lambda}(p_1, p_2) &= \frac{i}{4\pi}\frac{m}{|m|}\epsilon_{\mu\nu\lambda}\, e^{-\frac{i}{2}p_1\theta p_2}. && (13)
\end{aligned}$$

Substituting back to (8), we obtain the following fermion-loop induced term in S^{reg}_{eff},

$$S_{ind} = \pm\frac{1}{2} S_{CS} \tag{14}$$

with

$$S_{CS} = \frac{1}{4\pi}\int d^3x\, \epsilon^{\mu\nu\lambda}(g^2 A_\mu\partial_\nu A_\lambda + \frac{2i}{3} g^3 A_\mu * A_\nu * A_\lambda), \tag{15}$$

or in terms of the gauge field $\mathcal{A}_\mu = -igA_\mu$

$$S_{CS} = \frac{1}{4\pi}\int d^3x\, \epsilon^{\mu\nu\lambda}(\mathcal{A}_\mu\partial_\nu\mathcal{A}_\lambda + \frac{2}{3}\mathcal{A}_\mu * \mathcal{A}_\nu * \mathcal{A}_\lambda) \tag{16}$$

is the noncommutative Chern-Simons action. S_{CS} is local invariant under (7), while under a finite gauge transformation,

$$\mathcal{A}_\mu \to h^{-1}_* * \mathcal{A}_\mu * h + h^{-1}_* * \partial_\mu h \tag{17}$$

S_{CS} changes as [1]

$$S_{CS} \to S_{CS} - 2\pi w \tag{18}$$

where

$$S_{WZW} := \frac{1}{24\pi^2}\int_B d^3x\, \epsilon^{\mu\nu\lambda}(h^{-1}_* * \partial_\mu h * h^{-1}_* * \partial_\nu h * h^{-1}_* * \partial_\lambda h) \tag{19}$$

[1] We have dropped a total derivative piece $d(A * dh * h^{-1}_*)$ which on a manifold with boundary will give rises to an anomaly for the boundary theory. Descent relations and the algebraic structures of chiral anomaly still hold generally for a noncommutative gauge theory [27]. See also the third and fourth references of [26] for recent discussion.

is the noncommutative WZ term over B (or WZW action over ∂B) and $w := S_{WZW}$ for $B = S^3$ is the "winding number". Here h_*^{-1} is the inverse of h with respect to the Moyal product:

$$h * h_*^{-1} = h_*^{-1} * h = 1. \tag{20}$$

For any $h = e^{i\alpha}$ of $U(1)$, it is $h_*^{-1} = e^{-i\alpha}$. It is easy to check that w is invariant under an infinitesimal transformation $\delta h = \lambda * h$ and that under a finite transformation,

$$w(h' * h) = w(h') + w(h). \tag{21}$$

Now we claim that w is zero for the Abelian case. Consider

$$I[\alpha] = \epsilon^{\mu\nu\lambda} \int d^3x \; e^{-i\alpha} * \partial_\mu e^{i\alpha} * e^{-i\alpha} * \partial_\nu e^{i\alpha} * e^{-i\alpha} * \partial_\lambda e^{i\alpha}. \tag{22}$$

It is clear from (21) that $I[\frac{m}{n}\alpha] = \frac{m}{n} I[\alpha]$ for any integers m, n, therefore $I[s\alpha] = sI[\alpha]$ for any real constant s, and hence

$$I[\alpha] = \frac{\partial}{\partial s} I[s\alpha]. \tag{23}$$

But we already knew that I is invariant under arbitrary infinitesimal transformation, hence our claim. One can also get the same result by noticing that since the LHS of (23) is independent of s, one can evaluates the RHS of (23) at the particular value $s = 0$ and obtains the desired result. That $w = 0$ for $U(1)$ may be expected intuitively since S^3 is too big to fit in S^1. As a result, the 2-dimensional noncommutative WZW action [28] is well defined.

For the case of non-Abelian $U(N)$ gauge fields, we just have to replace (8) by

$$\begin{aligned} S_{eff}[A, m] &= \frac{1}{2}\mathrm{tr}(T^a T^b) \int \frac{d^3p}{(2\pi)^3} A_\mu^a(p) A_\nu^b(-p)(-i\Gamma^{\mu\nu}(p)) \\ &+ \frac{1}{3}\mathrm{tr}(T^a T^b T^c) \int \frac{d^3p_1}{(2\pi)^3} \frac{d^3p_2}{(2\pi)^3} A_\mu^a(p_1) A_\nu^b(p_2) A_\lambda^c(-p_1 - p_2)(-i\Gamma^{\mu\nu\lambda}(p_1, p_2)) \end{aligned} \tag{24}$$

with the same 2-point and 3-point functions. Thus one again obtains (14), now with

$$S_{CS} = \frac{1}{4\pi} \int d^3x \; \epsilon^{\mu\nu\lambda} \mathrm{tr}(\mathcal{A}_\mu \partial_\nu \mathcal{A}_\lambda + \frac{2}{3} \mathcal{A}_\mu * \mathcal{A}_\nu * \mathcal{A}_\lambda), \tag{25}$$

where $\mathcal{A}_\mu = -igA_\mu^a T^a$.

Again, S_{CS} is local gauge invariant, while under a finite gauge transformation (17) with h in $U(N)$, S_{CS} changes as

$$S_{CS} \to S_{CS} - 2\pi w \tag{26}$$

where

$$S_{WZW} := \frac{1}{24\pi^2} \epsilon^{\mu\nu\lambda} \int_B \mathrm{tr}(h_*^{-1} * \partial_\mu h * h_*^{-1} * \partial_\nu h * h_*^{-1} * \partial_\lambda h) \tag{27}$$

is the noncommutative WZ term over B and $w := S_{WZW}$ for $B = S^3$. It is easy to check that w is invariant under a local $U(N)$ gauge transformation $\delta h = \lambda * h$ and we have again (21) under a finite transformation. It is not clear whether w is an integer or not when integrated on S^3. One can shows that this number is independent of θ, at least to the first order in θ. Since $w[h^{*n}] = nw[h]$ and w is invariant under small changes of the map h, these suggest that w may again serve as some sort of homotopy invariant and that the θ-dependence factorize. If θ dependence does not disappear and w is not an integer, then one will need to arrange the fermions content so that the global anomaly cancel. We leave these interesting issues for future investigation.

2.2 Majorana fermions

We next consider the case of a Majorana spinor. The situation differs in that now nonplanar diagram can also contribute. Similar considerations have also been made in [20]. The Feynman rules have been worked out in [19] and we will not repeat them here. We note that the phase factor $e^{-iq_1 \frac{\theta}{2} q_2}$ for a Dirac spinor coupled to the gauge field (with momentum q_1, q_2, q_3 coming into the vertex) is now replaced by $-2i \sin(q_1 \frac{\theta}{2} q_2)$ due to the commutator nature of the coupling for the Majorana spinors. In particular the 2-point and 3-point functions are now given by

$$i\Gamma_M^{\mu\nu}(p) = -4g^2 \int \frac{d^3k}{(2\pi)^3} \mathrm{tr}[\gamma^\mu \frac{\not k - \not p - m}{(k-p)^2 + m^2} \gamma^\nu \frac{\not k - m}{k^2 + m^2}] \sin^2(\frac{\tilde{p}k}{2}), \tag{28}$$

$$\begin{aligned} i\Gamma_M^{\mu\nu\lambda}(p_1, p_2) = -8ig^3 \int \frac{d^3k}{(2\pi)^3} & \frac{\mathrm{tr}[\gamma^\mu(\not k - m)\gamma^\nu(\not k + \not p_2 - m)\gamma^\lambda(\not k - \not p_1 - m)]}{(k^2+m^2)((k+p_2)^2+m^2)((k-p_1)^2+m^2)} \\ & \cdot \sin(\frac{\tilde{p}_1 k}{2}) \sin(\frac{\tilde{p}_2 k}{2}) \sin(\frac{\tilde{p}_3 (k+p_2)}{2}), \end{aligned} \tag{29}$$

where for convenience we have denoted $\tilde{p} = p\theta$ and momentum conservation

$$p_1 + p_2 + p_3 = 0 \tag{30}$$

has to be used. Since

$$\sin^2(\frac{\tilde{p}k}{2}) = \frac{1}{2}(1 - \cos \tilde{p}k), \tag{31}$$

one can reduce (28) to a sum corresponding to planar and nonplanar contributions

$$\Gamma_M^{\mu\nu} = -2\Gamma_D^{\mu\nu} + \text{“nonplanar”}, \tag{32}$$

where “nonplanar” are expressions of the form

$$\int \frac{d^3k}{(2\pi)^3} \frac{f(k)}{((k-p)^2+m^2)(k^2+m^2)} e^{ik\tilde{p}} \tag{33}$$

and $f(k) = k^\mu k^\nu$, k^μ or 1. It is easy to show that the "nonplanar" terms all vanish in the large m limit. It is enough to calculate for the case of $f = 1$ as the others can be obtained by differentiating with respect to $\tilde{p}$. Introducing Schwinger parameters, one obtains

$$\begin{aligned}\int \frac{d^3k}{(2\pi)^3} \frac{1}{((k-p)^2+m^2)(k^2+m^2)} e^{ik\tilde{p}} &= \frac{1}{(2\sqrt{\pi})^3} \int_0^\infty dT \frac{1}{T^{1/2}} e^{-m^2T - \frac{\tilde{p}^2}{4T}} \int_0^1 dx e^{-Tx(1-x)p^2} \\ &< \int_0^\infty dT \frac{1}{T^{1/2}} e^{-m^2T-\frac{\tilde{p}^2}{4T}} \quad \sim \frac{1}{m} e^{-|m||\tilde{p}|} \qquad (34)\end{aligned}$$

and hence

$$\lim_{m\to\infty} \text{“nonplanar”} = 0 \qquad (35)$$

as long as $|\tilde{p}| = |\theta|\sqrt{p_1^2+p_2^2} \neq 0$, which is of measure zero in the integration of (8). Similarly one gets using (30)

$$i \sin(\frac{\tilde{p}_1 k}{2}) \sin(\frac{\tilde{p}_2 k}{2}) \sin(\frac{\tilde{p}_3(k+p_2)}{2}) = \frac{i}{4} \sin(\frac{p_1\theta p_2}{2}) + \text{phases involving internal momenta.} \qquad (36)$$

Therefore again we can write (29) as

$$\Gamma_M^{\mu\nu\lambda} = -2\Gamma_D^{\mu\nu\lambda} + \text{“nonplanar''} \qquad (37)$$

and similarly show that the "nonplanar" terms vanish in the large m limit. Substituting back to (8) and remember that there is now an extra factor of 1/2 since we are considering Majorana spinor and hence $S_{eff} = 1/2 \mathrm{tr} \log(i\not{D} + \not{A})$. We obtain finally the induced term

$$S_{ind} = \pm\frac{1}{2} S_{CS}. \qquad (38)$$

We note that the factor of 1/2 and 1/4 in (31) and (36) can be easily understood. There are one planar and one nonplanar diagram contributing to the 2-point function, and one planar and three nonplanar diagrams contributing to the 3-point function (given a definite cyclic order of the external momentum). We also note that in the commutative case, Majorana spinors are neutral and there is no Chern-Simons action induced from them. In the noncommutative case, a coupling can be written down that reduces to zero in the commutative limit. Both planar and nonplanar diagrams contribute to the effective actions, but the nonplanar diagrams are suppressed and only the planar diagrams survive the large m limit. Therefore upon removing the Pauli-Villars regulator, we are left with the induced Chern-Simons action (38).

For the $U(N)$ case, the coupling is replaced by

$$-ig(f^{abc}\cos(q_1\frac{\theta}{2}q_2) + d^{abc}\sin(q_1\frac{\theta}{2}q_2)) \qquad (39)$$

where we have adopted the normalization $\mathrm{tr}T^aT^b = \delta^{ab}/2$ and $T^aT^b = \frac{1}{2}(if^{abc}T^c + d^{abc}T^c)$. The $U(1)$ generator is $T^0 = \frac{1}{\sqrt{2N}}1$ and it is $f^{000} = 0$ and $d^{000} = \sqrt{2/N}$. The previous $U(1)$ case is recovered by $gd^{000} \to 2g$. Using the identities

$$-f^{ab'a'}f^{bc'b'}f^{ca'c'} = f^{ab'a'}d^{bc'b'}d^{ca'c'} = \frac{N}{2}f^{abc}, \tag{40}$$

$$d^{ab'a'}d^{bc'b'}d^{ca'c'} = \frac{N}{2}d^{abc} + \delta^{ab}\mathrm{tr}T^c + \delta^{bc}\mathrm{tr}T^a + \delta^{ca}\mathrm{tr}T^b \tag{41}$$

$$f^{ab'a'}f^{bc'b'}d^{ca'c'} = -\frac{N}{2}d^{abc} - \delta^{ab}\mathrm{tr}T^c + \delta^{bc}\mathrm{tr}T^a + \delta^{ca}\mathrm{tr}T^b \tag{42}$$

where T^a are in the defining representation. One easily obtains the induced Chern-Simons term (38), with now S_{CS} (25) defined in the adjoint representation.

Finally we remark that all of the above can be generalized straightforwardly to higher dimensions (provided that massive Majorana spinors exist in that dimensions) and one still obtains the same shift with the corresponding higher dimensional Chern-Simons action.

3 Discussions

In this paper, we investigated the induced Chern-Simons action due to Dirac and Majorana fermion coupled to Abelian $U(1)$ or non-Abelian $U(N)$ gauge fields. The surprising result is that for the Majorana spinor case, the induced term does not go to zero as $\theta \to 0$ and displays a discontinuity. Since there is a finite difference no matter how small is the noncommutativity, this kind of discontinuity in a physical quantity may be useful to searching of experimental signals of noncommutativity and Lorentz symmetry breaking in nature.

Similar θ-singularities have also been discovered in scalar field theories [16, 17, 18] and QED [19, 20]. These examples show that although classically noncommutative physics is a smooth deformation of the commutative description, there can be important new quantum mechanical terms that do not vanish in the commutative limit. The commutative limit and the classical limit generally do not commute. The technical reason is very simple, generally one cannot exchange the order of taking the $\theta \to 0$ limit and the integration.

One may then get the opposite impression that noncommutative quantum physics always do not have a smooth commutative limit. The example of the induced Chern-Simons term for Dirac spinors shows that this is not true. So long as only planar diagrams contribute, there is a smooth limit. Take another example, the computation of the chiral anomaly for Weyl fermion. Again only planar diagrams contribute and so the loop effects get modified only by a phase factor which depend on the external momentum. Therefore it can be Fourier transformed back easily and gives the usual expression of the anomaly, except that now the products are replaced by the Moyal products. There is again a smooth limit because only planar diagrams contribute.

The opposite situation would be more interesting: is there a case where only nonplanar diagrams contribute? From the string point of view, we need a string theory with only nonplanar worldsheets. One needs to identify a limit so that only nonplanar worldsheets survive. Consistency may be an important issue, but it may also be possible that the field theory limit is not sensitive to the details of the string consistency issues.

It is known that in the commutative case, the Chern-Simons action can also be induced from the gauge boson loops. This bosonic shift has been shown explicitly in the $(2+1)$-dimensional case by Witten [29] using a saddle point analysis. The situation is however less clear if one perform a perturbative calculation and the shift depends on the regularization scheme. A natural regularization is to use string theory. It would be interesting [30] to see what string regularization has to say about the shift for both the commutative and noncommutative case.

Another interesting question concerns the nonrenormalization of our result. In the commutative case, it has been shown through explicit calculations that the fermion induced Chern-Simons term is not renormalized at two loops [31], for both the Abelian and non-Abelian case. Beyond this result, we have the Coleman-Hill theorem [32] for the Abelian case which shows that there is no contribution to the induced Chern-Simons action beyond one loop. Due to the topological origin of the non-Abelian Chern-Simons action and its connection with the chiral anomaly, it is also expected that it should not be renormalized beyond one-loop. Thus the situation is more or less clear. It would be interesting to investigate the status of nonrenormalization theorem for induced Chern-Simons action, as well as for chiral anomaly.

Induced Chern-Simons term in odd dimensions were originally [2, 3] obtained for fermions coupled to gauge fields in a flat spacetime. This has been extended [33] to full generality for arbitrary curved backgrounds and any odd dimensions and is shown to be related to the Atiyah-Patodi-Singer index theorem. The induced Chern-Simons action in $(2n+1)$-dimensions is given (up to a normalization factor) by the secondary characteristic class $Q(A,\omega)$ satisfying

$$dQ(A,\omega) = \hat{A}(R)ch(F)|_{2n+2}, \tag{43}$$

where ω is the gravitational connection. In view of the results [26] on the mathematical structure of anomalies in noncommutative gauge theory, it seem appropriate to investigate the deformation theory for elliptic operators due to a Moyal product and to study the possible "topological" meaning of the Chern-Simons action and chiral anomaly and to establish the corresponding index theorems. Better understanding of the properties of the noncommutative WZW action and its topological meaning is also highly desired.

It would also be interesting to investigate aspects of AdS/CFT correspondence [22, 34] in relation to chiral anomaly and Chern-Simons action. This will provide another channel to understand better this so far rather poorly understood correspondence. We plan to come back to these issues in the future.

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